Phase Noise in Oscillators—Leeson Formula Revisited

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***Abstract*—In the field of linear feedback-systems formalism, the Leeson formula is a useful tool for the determination of phase noise in feedback oscillators.**

Nevertheless, a direct application of the Leeson model without care can lead to erroneous results because the formula contains hidden parameters that are generally unwittingly ill evaluated or neglected. Thus, a brute-force calculation of phase noise with the Leeson formula can lead to errors of several orders of magnitude (i.e., several tens of decibels).

A detailed analysis enables us to enlighten the hidden parameters leading to a modified Leeson formula that is valid for all oscillator circuits. It explicitly takes into account all the parameters needed for phase-noise calculation.

In order to demonstrate the ease of use and accuracy of the new formula, we apply it to several oscillator circuits with lumped elements, transmission lines, and high-Q resonators. Finally the analytical results are confirmed by numerical simulations with a nonlinear transistor model.

***Index Terms*—Energy stored, Leeson formula, loop gain, oscillation conditions, oscillators, oscillator Q factor, passive circuit Q factor, phase of loop gain, phase noise, slope factor.**

I. INTRODUCTION

IN THE FIELD of linear feedback systems formalism, the Leeson formula is a useful tool for the determination of phase noise in feedback oscillators [1]. However, a successful application requires a careful identification of the parameters included in the formula according to the oscillator structure.

Fig. 1 shows a conventional representation of a feedback oscillator. It includes the following:

- on the one hand, the amplifying device: the transistor;
- on the other hand, the feedback path, which includes the load conductance, and permits to feedback a small part of the output signal to the input of the amplifying device through a selective filter.

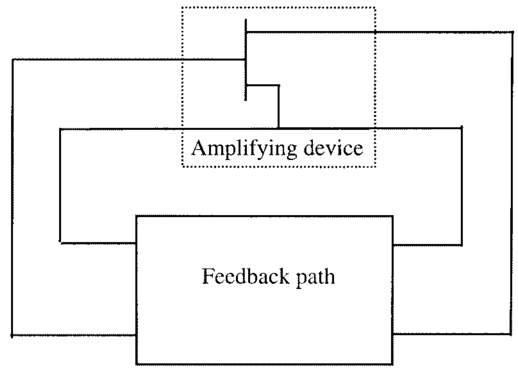

Fig. 1. General representation of a feedback oscillator.

Fig. 2 shows a linear representation of the feedback oscillator in the frequency domain. For phase-noise calculation purposes, a carrier voltage of peak value V_{01} at the oscillation frequency ω_0 is implied at the controlling input port of the transistor.

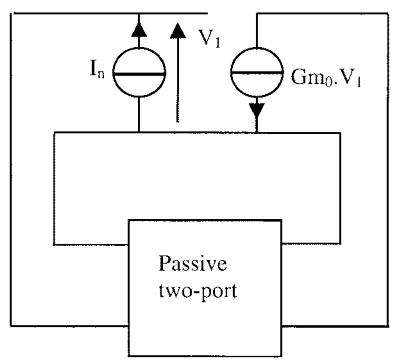

Fig. 2. Linear representation of oscillator circuit in the frequency domain.

The amplifying (active) function of the transistor is highlighted: the voltage-controlled current source of the transistor $G_{M0}V_1$ is isolated from the passive elements of the transistor model, which are now included in the passive reciprocal feedback path. Through a linear calculation in the frequency domain: 1) G_{M0} is the conventional positive transconductance of the transistor and 2) the transistor noise is represented by an input white noise source $I_n(\omega_n)$. It must be noted that the linearization implies that the nonlinear elements are approximated by their equivalent values calculated for the oscillation amplitude at the oscillation frequency, as described in [2, Ch. 2].

In the frequency domain, the noise source $I_n(\omega_n)$ is such that $\langle I_n(\omega_n) I_n(\omega_n)^* \rangle$ represents by definition the average power dissipated by the noise current source in unit resistance and unit bandwidth [3] [4] centered at angular frequency ω_n.

Then [5] $\left\langle |I_n|^2 \right\rangle = S_{I_n}\Delta f$ with $\Delta f = 1$ Hz.

Finally,

$$\left\langle |I_n|^2 \right\rangle_{1\,\text{Hz}} = S_{I_n} \tag{1}$$

Note that the units of $\left\langle |I_n|^2 \right\rangle$ are A^2, but $\left\langle |I_n|^2 \right\rangle$ has the same numerical value that the power spectral density S_{I_n} (with units: A^2 per Hz).

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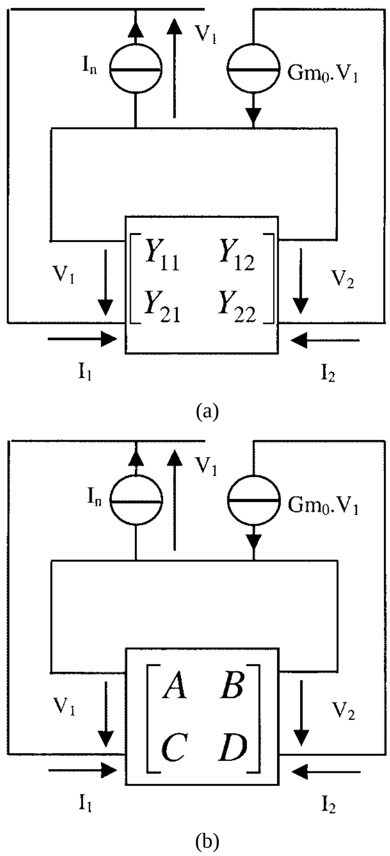

Fig. 3. (a) $[Y]$ representation of the feedback circuit. (b) $[C]$ chain-matrix representation of the feedback circuit.

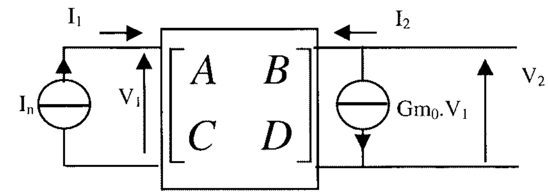

Fig. 4. Feedback oscillator with a chain-matrix description of the passive circuit.

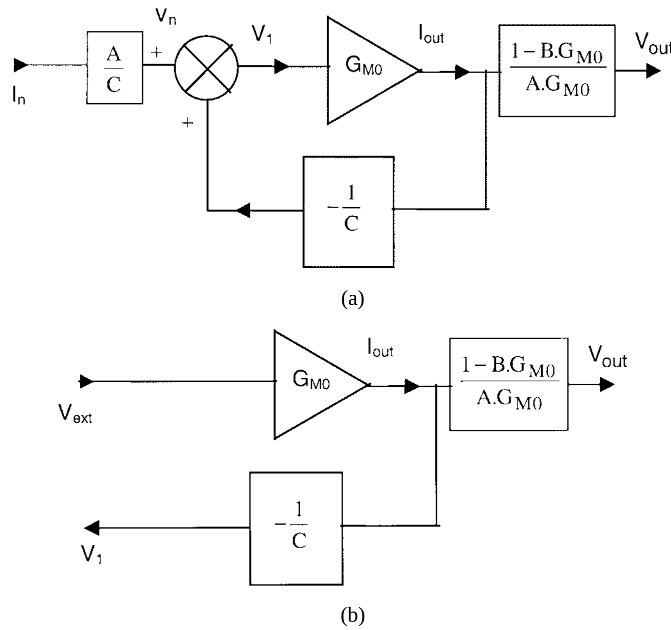

Fig. 5. (a) Closed-loop representation of the oscillator circuit. (b) Open-loop transfer function calculation by Bode formalism.

One noise source alone is not always sufficient to characterize a noisy transistor.

However, in the most general case, two correlated sources $\mathrm{In1_{input}}$ and $\mathrm{In2_{input}}$ are sufficient.

$\left\langle |\mathrm{in}|^2 \right\rangle$ becomes

$$\left\langle |\mathrm{In}|^2 \right\rangle = \left\langle |\mathrm{In1_{input}}|^2 \right\rangle + \left\langle |\mathrm{In2_{input}}|^2 \right\rangle + 2\Re\left(\mathrm{In1^*_{input}} \mathrm{In2_{input}}\right).$$

For phase-noise calculation, this input noise source must be carefully evaluated according to the localization of the physical noise sources in the transistor.

Fig. 3(a) and (b) is two equivalent electrical circuits of the feedback oscillator.

Unfortunately, the Y matrix does not exist for all the passive circuits so the chain matrix (also called the $ABCD$ matrix), which can always be evaluated, will be used. The matrix equation of Fig. 4 is written as follows:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \tag{2}$$

with $I_1 = I_n$ and $-I_2 = G_{M0}.V_1$

Leeson [1], using a single resonator feedback network, has derived the following formula:

$$S_{\Delta\varphi_{\mathrm{out}}} = S_{\Delta\varphi_{\mathrm{in}}} \left[1 + \left(\frac{\omega_0}{2Q_{\mathrm{Loscill}}}\right)^2 \frac{1}{\Delta\omega^2}\right]. \tag{3}$$

The parameters of (3) must be now identified in order to apply it to oscillators described by the equivalent circuit of Fig. 4 and (2).

II. Determination of $S_{\Delta\varphi_{\mathrm{in}}}$

In the field of linear feedback system formalism, the equivalent closed-loop representation of Fig. 5(a) is deduced. In this figure, we note $G_{M0} \cdot V_1 = I_{\mathrm{out}}$, $V_2 = V_{\mathrm{out}}$; V_n represents the equivalent input noise voltage source due to I_n.

In order to determine $S_{\Delta\varphi_{\mathrm{in}}}$, we first calculate V_n around the oscillation frequency ω_0, which will be determined later. Let $\omega_n = \omega_0 + \Delta\omega$. From Fig. 5(a), we obtain

$$V_n\big|_{\omega_n} = I_n \frac{A}{C} \approx I_n \frac{A_0}{C_0}$$

where A_0 and C_0 are the chain-matrix coefficients taken at ω_0 and

$$\left\langle |V_n|^2 \right\rangle = \left\langle |I_n|^2 \right\rangle \left|\frac{A_0}{C_0}\right|^2. \tag{4}$$

Note that the approximation used in (4) is valid because the transfer function A/C is not included in the loop. It does not participate in the positive feedback. Moreover, the phase noise is calculated for small frequencies offset from carrier.

The input phase noise spectral density $S_{\Delta\varphi_{\mathrm{in}}}$ can now be determined.

Let us look at two narrow-band (1 Hz) uncorrelated components V_{n-} and V_{n+}, respectively, located at angular frequencies $\omega_0 - \Delta\omega$ and $\omega_0 + \Delta\omega$.

A classical treatment [6], [7] shows that by addition to a pure carrier signal of peak value V_{01} at the frequency ω_0, these uncorrelated components give rise to a modulated carrier with a phase-noise spectral density $S_{\Delta\varphi_{\rm in}}$, which is given at an offset frequency $\Delta\omega$ from the carrier by

$$S_{\Delta\varphi_{\rm in}(\Delta\omega)} = 2\,\frac{\left\langle |V_n|^2 \right\rangle}{|V_{01}|^2}. \tag{5}$$

By taking into account (1) and (4), and noting that I_n is a white noise source, we then obtain a white phase-noise spectral density in a 1-Hz bandwidth[1]

$$S_{\Delta\varphi_{\rm in}} = 2\,\frac{\left\langle |I_n|^2 \right\rangle}{|V_{01}|^2}\left|\frac{A_0}{C_0}\right|^2 = 2\,\frac{S_{I_n}}{|V_{01}|^2}\left|\frac{A_0}{C_0}\right|^2. \tag{6}$$

In order to simplify the notation in the following calculations, we define for convenience $\Delta\varphi_{\rm in} = \sqrt{S_{\Delta\varphi_{\rm in}}}$. Note that only $S_{\Delta\varphi_{\rm in}}$ has a physical significance, $\Delta\varphi_{\rm in}$ is not useful in itself.

The output transfer function is also calculated near the oscillation frequency. Using the same approximations as for (4), we can derive from Fig. 5(a)

$$H_{\rm out}\big|_{\omega_0+\Delta\omega} = \frac{1 - B\,G_{M0}}{AG_{M0}} \approx \frac{1 - B_0 \cdot G_{M0}}{A_0 G_{M0}}. \tag{7}$$

Note that if the output transfer function is a function of frequency near ω_o, AM/PM noise conversion may take place.

III. Determination of the Oscillation Frequency

In order to find the oscillation frequency, we calculate the open-loop gain according to the Bode method [9]. By setting $I_n = 0$ and opening the feedback loop, Fig. 5(b) is obtained.

We have

$$G_{M0} V_{\rm ext} = I_{\rm out} \text{ and } -\frac{I_{\rm out}}{C} = V_1. \tag{8}$$

The open-loop gain can be written as

$$\tilde{G} = \frac{V_1}{V_{\rm ext}} = -\frac{G_{M0}}{C} \tag{9}$$

where $\tilde{G}$ denotes the complex voltage gain in the frequency domain.

[1] In the technical literature, the phase noise (5) or (6) is called the "phase noise spectral density." Nevertheless, physically and dimensionally it is a power ratio as is demonstrated below.
From (1), $\langle |I_n|^2 \rangle = S_{I_n}\Delta f$ with S_{I_n}: spectral power density, and $\Delta f = 1$ Hz.
$\langle |I_n|^2 \rangle$, which is numerically equal to S_{I_n}, is then proportional to a noise power in a 1-Hz bandwidth.
On the other hand, $(|V_{01}|^2 \,.\, |C_0|^2)/2$ is proportional to the carrier power, and $|A_0|^2$ is a number. $S_{\Delta\varphi_{\rm in}}$ is then a power ratio, in a 1-Hz bandwidth, and is not a spectral density. This misleading denomination of $S_{\Delta\varphi}$ (or that of script $\mathcal{L}(\omega) = S_{\Delta\varphi}/2$) has been pointed out for a long time past, e.g., by Kaertner in [8]. However, today, the term "phase noise spectral density" prevails in rad^2 in 1 Hz or in rad^2/Hz.

Oscillation conditions are fulfilled for $V_1 = V_{\rm ext}$. It follows that

$$\tilde{G}(\omega_0) = 1 = -\frac{G_{M0}}{C} = -\frac{G_{M0}}{C_0}. \tag{10}$$

Note that from (10), C_0 must be real and negative.

Let $C = C_R + jC_I$, the oscillation frequency is determined by

$$C_I(\omega_0) = 0 \tag{11}$$

and then $C_R(\omega_0) = C_0 = -G_{M0}$

IV. Determination of the "Loaded Q Factor of the Oscillator"

From (9), the loop gain is written as

$$\tilde{G} = |G|\,e^{j\cdot\varphi} = -\frac{G_{M0}}{C}. \tag{12}$$

At the oscillation frequency ω_0

$$|G|\cdot(\omega_0) = 1$$
$$\varphi(\omega_0) = 0.$$

From (11) and (12), the phase slope of the loop gain can be obtained at the oscillation frequency

$$\left.\frac{d\varphi}{d\omega}\right|_{\omega_0} = -\frac{C'_I}{C_0} \tag{13}$$

with $C'_I = dC_I/d\omega|_{\omega_0}$. We denote the "oscillator loaded Q factor" $Q_{\rm Loscill}$ as

$$Q_{{\rm Loscill}(\omega_o)} = \frac{\omega_o}{2}\left|\frac{d\varphi}{d\omega}\right|\Bigg|_{\omega_o} = \frac{\omega_o}{2}\left|\frac{-C'_I}{C_0}\right|. \tag{14}$$

This expression, always positive, is applicable to all feedback networks independently of the parallel or series tuned nature of the feedback tank.

It must be noted as a general rule that the "oscillator loaded Q factor" $Q_{\rm Loscill}$ does not coincide with the loaded Q factor of the passive circuit. It turns out that these two coefficients coincide for some elemental feedback networks. However, as a general rule, they are different. This difference can reach one or several orders of magnitude.

The relevant coefficient for the calculation of the output phase noise $Q_{\rm Loscill}$ is proportional to the group delay of the feedback path. It is directly related to the phase-frequency relationship of the oscillator loop gain, and can be calculated from (9) (12), and (14).

Note that the difference between the loaded Q factor of the passive circuit alone and the "oscillator loaded Q factor" has been pointed out for one-port negative resistance oscillators in [10].

V. Determination of the Output Phase Noise

For the output phase-noise determination purposes, the normalized representation of the feedback oscillator shown in Fig. 6 is deduced from (6), (7), and (10)–(14).

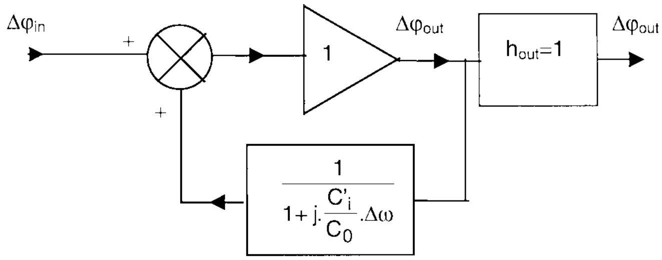

Fig. 6. Normalized closed-loop representation of a feedback oscillator for phase noise calculations.

A straightforward calculation gives successively

$$\Delta\varphi_{\rm in} = \Delta\varphi_{\rm out}\left(1 - \frac{1}{1 + j\,\dfrac{d\varphi}{d\omega}\cdot\Delta\omega}\right) \tag{15}$$

$$\Delta\varphi_{\rm in} = \Delta\varphi_{\rm out}\left(1 - \frac{1}{1 + j\,\dfrac{C_I'}{C_0}\cdot\Delta\omega}\right) \tag{16}$$

$$\Delta\varphi_{\rm out} = \Delta\varphi_{\rm in}\left(1 - j\,\frac{C_0}{C_I'\cdot\Delta\omega}\right) \tag{17}$$

$$S_{\Delta\varphi_{\rm out}} = S_{\Delta\varphi_{\rm in}}\left(1 + \left(\frac{C_0}{C_I'}\right)^2\frac{1}{\Delta\omega^2}\right). \tag{18}$$

Finally, from (6),

$$S_{\Delta\varphi_{\rm out}} = 2\,\frac{\left\langle |I_n|^2\right\rangle}{|V_{01}|^2}\left|\frac{A_0}{C_0}\right|^2\left(1 + \left(\frac{C_0}{C_I'}\right)^2\frac{1}{\Delta\omega^2}\right). \tag{19}$$

Equations (6), (10), (11), (14), and (19) constitute the main results of this paper.

Near carrier frequency, $S_{\Delta\varphi_{\rm out}}$ becomes

$$S_{\Delta\varphi_{\rm out}} = 2\,\frac{\left\langle |I_n|^2\right\rangle}{|V_{01}|^2}\left|\frac{A_0}{C_I'}\right|^2\frac{1}{\Delta\omega^2}. \tag{20}$$

Equations (19) and (20) must now be compared to the corresponding Leeson formulas recalled below for convenience as follows:

$$S_{\Delta\varphi_{\rm out}} = S_{\Delta\varphi_{\rm in}}\left(1 + \left(\frac{\omega_0^2}{4Q_{\rm Loscill}^2\Delta\omega^2}\right)\right) \tag{21}$$

and near carrier frequency

$$S_{\Delta\varphi_{\rm out}} = S_{\Delta\varphi_{\rm in}}\left(\frac{\omega_0^2}{4Q_{\rm Loscill}^2\Delta\omega^2}\right). \tag{22}$$

We are now in position to discuss and compare the different formulas by applying them to several oscillator circuits.

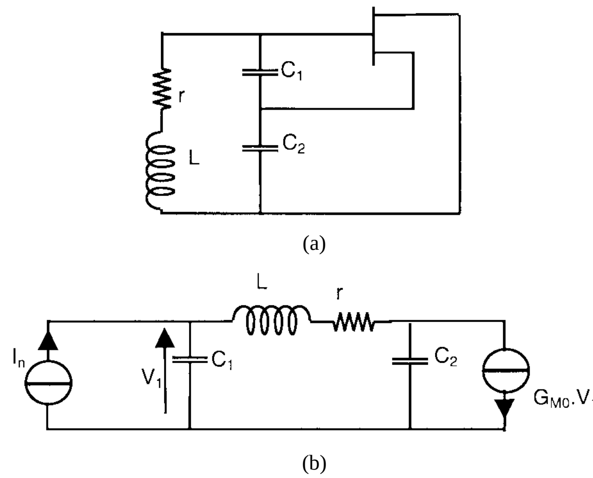

Fig. 7. (a) Colpitts oscillator: the ground node can be placed anywhere. (b) Electrical equivalent circuit.

VI. Application—Discussion

A Simple Example: Colpitts Oscillator

As a first example, the conventional well-known Colpitts oscillator is analyzed. Fig. 7(a) and (b) shows the electrical circuit.

A straightforward calculation gives the terms A and C of the chain matrix

$$A = 1 - LC_2\omega^2 + jrC_2\omega$$
$$C = j\,(C_1+C_2)\,\omega\left[1 - \frac{L\cdot C_1C_2}{C_1+C_2}\,\omega^2 + j\,\frac{rC_1C_2}{C_1+C_2}\,\omega\right]. \tag{23}$$

By neglecting $rC_1\omega \ll 1$, we find the following successively.

- The oscillation frequency

$$\omega_0 = \sqrt{\frac{C_1+C_2}{LC_1C_2}}. \tag{24}$$

- The coefficient A_0

$$A_0 = -\frac{C_2}{C_1}. \tag{25}$$

- The coefficient C_0

$$C_0 = -r\,\frac{C_1+C_2}{L}. \tag{26}$$

- The coefficient $C_I' = dC_I/d\omega|_{\omega_0}$

$$C_I' = -2(C_1+C_2). \tag{27}$$

From (19), we obtain

$$S_{\Delta\varphi_{\rm out}} = 2\,\frac{\left\langle |I_n|^2\right\rangle}{|V_{01}|^2}\,\frac{L^2C_2^2}{r^2C_1^2(C_1+C_2)^2}\left[1 + \frac{r^2}{4L^2\Delta\omega^2}\right] \tag{28}$$

and near the carrier frequency

$$S_{\Delta\varphi_{\rm out}} = 2\frac{\left\langle |I_n|^2\right\rangle}{|V_{01}|^2}\,\frac{1}{C_1^2\left(\dfrac{C_1}{C_2}+1\right)^2}\,\frac{1}{\Delta\omega^2}. \tag{29}$$

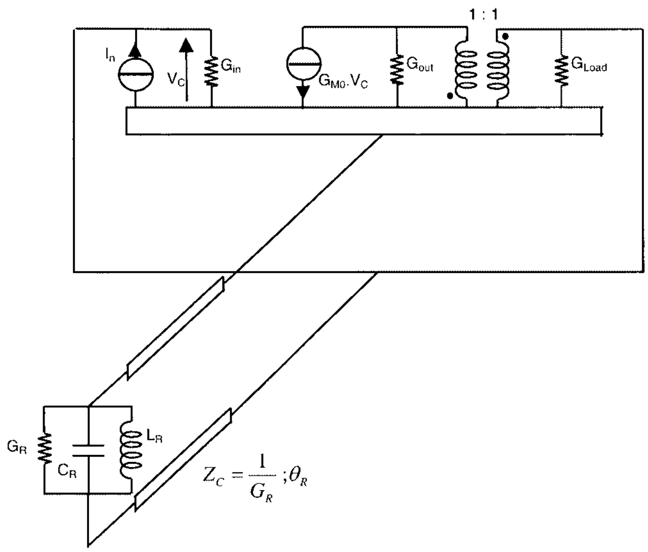

Fig. 8. Feedback oscillator with a transmission-line coupled resonator.

Equation (29) clearly shows that, by increasing C_1, the output phase noise is decreased.

At high frequencies, C_1 generally reduces to the input capacitance of the transistor alone. The ratio $\left\langle |I_n|^2 \right\rangle / C_1^2$ is then inversely proportional to the transistor area and, accordingly, the output phase noise is also inversely proportional to the transistor area.

It must be noted that (29) can be easily evaluated by designers.

If we now write the Q factor of the passive circuit as

$$Q_L = \frac{L \cdot \omega_0}{r}$$

and

$$S_{\Delta\varphi_{\text{in}}} = 2\,\frac{\overline{\left\langle |I_n|^2 \right\rangle}}{|V_{01}|^2}\,\frac{L^2 C_2^2}{r^2 C_1^2 (C_1 + C_2)^2}$$

it follows from (28) and (29) that

$$S_{\Delta\varphi_{\text{out}}} = S_{\Delta\varphi_{\text{in}}}\,\frac{\omega_0^2}{4Q_L^2 \Delta\omega^2}.$$

In the Colpitts oscillator circuit, the "oscillator loaded Q factor" coincides with the loaded Q of the passive circuit.

Example of Evaluation of the Loaded Q_{Loscill} Factor of a Feedback Oscillator

From (14), we have

$$Q_{\text{Loscill}(\omega_o)} = \frac{\omega_o}{2}\left|\frac{d\varphi}{d\omega}\right|\Bigg|_{\omega_o} = \frac{\omega_o}{2}\left|\frac{-C_I'}{C_0}\right|.$$

The hidden parameters included in the Leeson formula may result in a Q_{Loscill} different from the loaded Q factor Q_L of the feedback tank by more than one order of magnitude.

In order to account for such a possible difference, let us look at the example of Fig. 8.

This example has been chosen for its simplicity. It allows a direct comparison between an analytical calculation and a numerical simulation. The schematic shows an oscillator circuit, which includes a dielectric resonator [11] coupled by means of a lossless transmission line of characteristic impedance $Z_C = 1/G_R$ and electrical length θ_R at the oscillation frequency. The purpose of introducing the line is to allow variation in the loaded Q factor of the oscillator (without simultaneously varying the resonant frequency or, to the first order, the loaded Q of the passive circuit) by the varying length.

The admittance brought back by the resonator and its coupling line is written as follows:

$$Y = G_R + j2C_R e^{-j2\theta_R}\Delta\omega.$$

The total admittance of the passive circuit becomes

$$Y_T = G_T + j2C_R e^{-j2\theta_R}\Delta\omega$$

with

$$G_T = G_R + G_{\text{in}} + G_{\text{out}} + G_{\text{load}}.$$

Then

$$\frac{d\varphi_{Y_T}}{d\omega} = \frac{2C_R\cos(2\theta_R)}{G_T}.$$

The loaded Q factor Q_L of the passive circuit can be written as

$$Q_L = \frac{\omega_o}{G_T}\left(C_R + \frac{G_R\theta_R}{\omega_o}\right).$$

Now, without loss of accuracy, the energy stored in the coupling line may be practically neglected as compared to that of the resonator, the loaded Q factor Q_L of the passive circuit alone may be approximated to the first order by

$$Q_L = \omega_o\,\frac{C_R}{G_T}.$$

The loaded Q factor of the oscillator becomes

$$Q_{\text{Loscill}(\omega_o)} = \frac{\omega_o}{2}\left|\frac{d\varphi_{Y_T}}{d\omega}\right| = Q_L\,|\cos(2\theta_R)|. \tag{30}$$

By application of (6) and (22), we obtain

$$S_{\Delta\varphi_{\text{out}}} = S_{\Delta\varphi_{\text{in}}}\left[1 + \left(\frac{\omega_0}{2Q_L\cos(2\theta_R)\Delta\omega}\right)^2\right]$$
$$\approx S_{\Delta\varphi_{\text{in}}}\left(\frac{\omega_0}{2Q_L\cos(2\theta_R)\Delta\omega}\right)^2 \tag{31}$$

with

$$S_{\Delta\varphi_{\text{in}}} = \frac{2\cdot\left\langle |I_n|^2\right\rangle}{G_T^2\,|V_c(\omega_o)|^2}.$$

A wrong application of the Leeson formula would have given

$$S_{\Delta\varphi_{\text{out}}\text{.WRONG}} = S_{\Delta\varphi_{\text{in}}}\left(\frac{\omega_0}{2Q_L\Delta\omega}\right)^2 \tag{32}$$

so that

$$\frac{S_{\Delta\varphi_{\text{out}}}}{S_{\Delta\varphi_{\text{out}}\text{WRONG}}} = \left(\frac{1}{\cos(2\theta_R)}\right)^2. \tag{33}$$

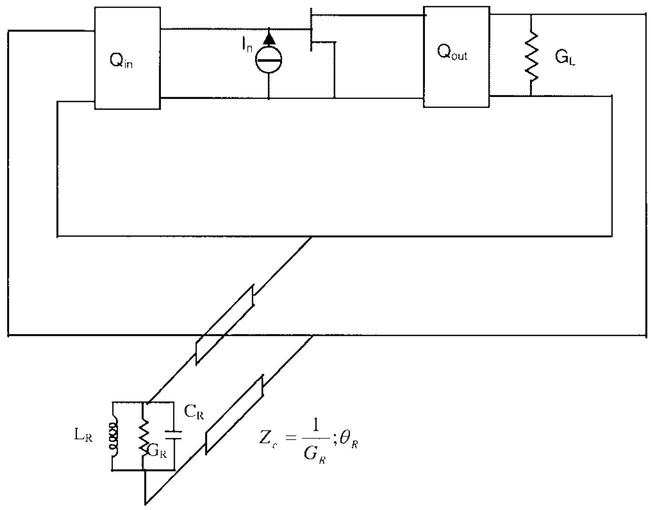

Fig. 9. Schematic of the transmission-line coupled resonator oscillator used for numerical simulations.

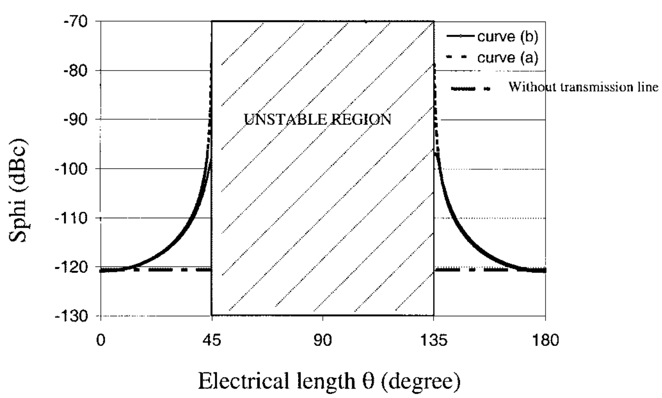

Fig. 10. Output phase-noise spectral density versus the electrical length θ_R.

We note first that $S_{\Delta\varphi_{\text{outWRONG}}}$ is optimistic. Besides, when $2.\theta_R$ tends to $\pi/2$, the ratio tends to $+\infty$. In this example, the loaded Q factor of the passive circuit is different from the "loaded Q factor of the oscillator." It would be an error to confuse these two quantities when evaluating the oscillator phase noise.

Numerical Simulation

Fig. 9 shows the circuit simulated with a full nonlinear model of a high electron-mobility transistor (HEMT). The output phase noise is calculated at the load G_L.

Simulations were performed with a nonlinear frequency-domain simulator [12].

A stability analysis of the steady-state operating point was first performed. It showed that, for stable operation, one must have $\cos(2.\theta_R) > 0$.

Noise analysis was then attempted. Fig. 10 shows the output phase-noise density obtained at 100 kHz of frequency offset from the carrier for a white noise input source. Besides as expected, simulations show that the phase noise rolloff as a function of the frequency offset from carrier is -20 dB/decade.

Fig. 10 speaks for itself and confirms the analytical results of (20) and (31).

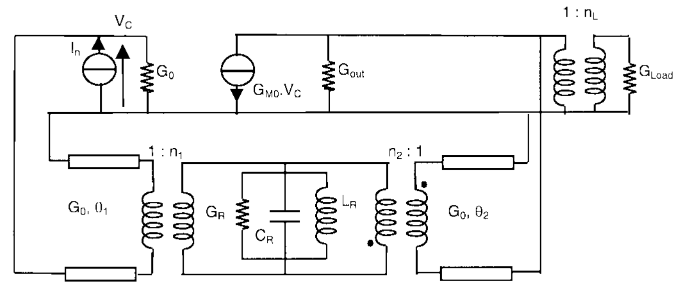

Fig. 11. Feedback oscillator with a resonator coupled in transmission between two transmission lines.

VII. Example of Determination of the Input Phase Noise in a Feedback Oscillator

The input phase-noise spectral density is written from (6) as follows:

$$S_{\Delta\varphi_{\text{in}}} = 2\,\frac{\overline{\left\langle |V_n|^2 \right\rangle}}{|V_{01}|^2} = 2\,\frac{\overline{\left\langle |I_n|^2 \right\rangle}}{|V_{01}|^2}\left|\frac{A_0}{C_0}\right|^2.$$

For a fixed "oscillator loaded Q factor," the output phase-noise spectral density varies as $|A_0/C_0|^2$, which is a function of the feedback path configuration.

As an example, let us consider the oscillator circuit of Fig. 11.

In the proposed configuration, the dielectric resonator is coupled in transmission between two lossless transmission lines whose total electrical length is equal to $2.\pi$ at the oscillation frequency.

In order to simplify the analytical calculations, we assume that, in Fig. 11, the input conductance of the transistor, as well as the sum $G_{\text{out}} + n_L^2\, G_{\text{load}}$ are equal to G_0. Moreover, the admittance presented by the feedback circuit at the transistor output port is also equal to G_0 at the oscillation frequency.

The parameters A, C, C_I' of the chain matrix representing the feedback circuit are

$$\begin{aligned} A = &-\frac{1}{2}\left[\frac{n_2}{n_1}+\frac{n_1}{n_2}+\left(\frac{n_2}{n_1}-\frac{n_1}{n_2}\right)\cos(2\theta_1)\right.\\ &+ n_1 n_2\,\frac{G_R}{G_o}\left(1-\cos(2\theta_1)\right)\\ &- \frac{n_1 n_2}{G_0}\,\frac{dB_R}{d\omega}\sin(2\theta_1)\,\Delta\omega\\ &+ j\left[\sin(2\theta_1)\left(\frac{n_1}{n_2}-\frac{n_2}{n_1}+n_1 n_2\,\frac{G_R}{G_o}\right)\right.\\ &\left.\left.+\frac{n_1 n_2}{G_0}\,\frac{dB_R}{d\omega}\left(1-\cos(2\theta_1)\right)\Delta\omega\right]\right] \end{aligned} \tag{34}$$

$$C = -\left[G_0\left(\frac{n_2}{n_1}+\frac{n_1}{n_2}\right)+n_1 n_2\left(G_R + j\,\frac{dB_R}{d\omega}\Delta\omega\right)\right] \tag{35}$$

$$C_I' = -n_1 n_2\,\frac{dB_R}{d\omega}. \tag{36}$$

Note that A, C, and C_I' were calculated around the resonant frequency ω_0 of the resonator for $\theta_1 + \theta_2 = 2\pi$ at ω_0.

When oscillation conditions are satisfied, we have

$$\omega_{\text{oscillation}} = \omega_0$$
$$C_0 = -\left[G_0\left(\frac{n_2}{n_1}+\frac{n_1}{n_2}\right)+n_1 n_2 G_R\right] = -2G_0\sqrt{G_p} \tag{37}$$

with

$$n_1^2 = \frac{G_0}{G_R}(G_p - 1) \tag{38}$$
$$n_2^2 = \frac{G_0}{G_R}\left(1-\frac{1}{G_p}\right) \tag{39}$$
$$G_p = \frac{G_{M0}^2}{4G_0^2}. \tag{40}$$

It must be noted that, as expected, the oscillation condition (37) is independent of θ_1. On the other hand, the loaded Q factor Q_{Loscill} of the feedback oscillator is also independent of θ_1. From (14), we have

$$Q_{\text{Loscill}} = \frac{\omega_0}{4G_R}\,\frac{G_p - 1}{G_p}\,\left.\frac{dB_R}{d\omega}\right|_{\omega_0}. \tag{41}$$

However, the term A_0 depends on θ_1 by

$$A_0 = -\sqrt{G_p}\left[1+\frac{1-G_p}{G_p}\left(\cos(2\theta_1) - j\sin(2\theta_1)\right)\right]. \tag{42}$$

It follows that

$$S_{\Delta\varphi_{\text{in}}} = \frac{\overline{\left\langle |I_n|^2\right\rangle}}{2G_0^2|V_{01}|^2}\left|1-\left(1-\frac{1}{G_p}\right)e^{-j2\theta_1}\right|^2. \tag{43}$$

The main source of error in the input phase-noise spectral density calculation comes from an erroneous evaluation of the input noise voltage V_n by taking loading admittances independently of the oscillator configuration. The most common error is to consider a matched input transistor.

In this latter case, an indiscriminate application of the Leeson formula would have given

$$S_{\Delta\varphi_{\text{in}}\text{WRONG}} = \frac{\overline{\left\langle |I_n|^2\right\rangle}}{2G_0^2|V_{01}|^2}.$$

Then

$$\frac{S_{\Delta\varphi_{\text{out}}}}{S_{\Delta\varphi_{\text{out}}\text{WRONG}}} = \frac{S_{\Delta\varphi_{\text{in}}}}{S_{\Delta\varphi_{\text{in}}\text{WRONG}}} = \left|1-\left(1-\frac{1}{G_p}\right)e^{-j2\theta_1}\right|^2. \tag{44}$$

Equation (44) shows that an erroneous application can lead to an error of $\approx (2G_p)^2$.

Noting that G_p can reach up to 10 dB or more in oscillation, the output phase noise error can reach up to 26 dB or more.

A numerical simulation with a nonlinear model of an HEMT transistor including five nonlinearities has been performed on a fundamental frequency feedback oscillator and confirms the analytical linear previsions. The unloaded Q factor of the dielectric resonator was 3000 and the oscillation frequency was 10 GHz.

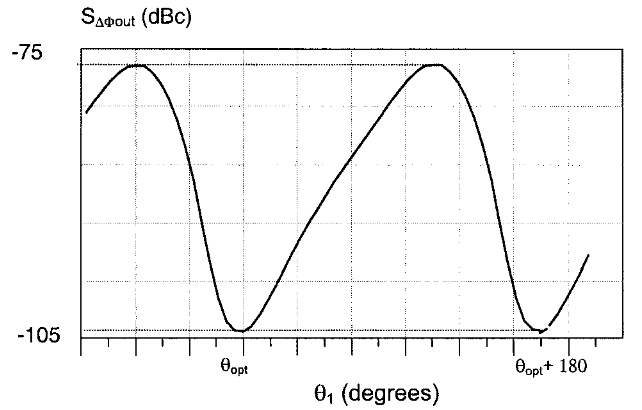

Fig. 12. Output phase-noise spectral density versus the electrical length θ_1 of a transmission line in the feedback path.

The power gain of the transistor was 13 dB in oscillation. Note that the power gain of the transistor must not be confused with the loop gain of the oscillator.

Fig. 12 shows the output phase-noise spectral density obtained as a function of θ_1 at 10 kHz of frequency offset from carrier for a white noise input current source.

As expected, we found that the oscillation frequency and output power were independent of θ_1 since the oscillation conditions (37) are independent of θ_1.

An important conclusion can be deduced from this example. For a fixed transistor, loaded Q factor of the oscillator, and input noise current source, the output phase noise can vary by over 20 dB depending only on the localization of the resonator in the feedback path, which accordingly induces different values of $S_{\Delta\varphi_{\text{in}}}$.

VIII. Influence of the Input Current Noise Source

In the Leeson model, the input current noise source I_n is a white noise source.

We will not discuss here the additional $1/f$ component, which can be added from measurement of the frequency corner f_c of the noise source I_n.

Nevertheless, in our previous calculations, the low frequencies, near dc, of the white noise source spectrum have been neglected, i.e., the up-conversion has not been taken into account.

Despite carefully designed bias circuits, the low-frequency spectrum of the white noise source contributes to the output phase noise generation.

In fact, the following three uncorrelated components of the white noise source I_n must be considered:

1) lower sidebands near the carrier;
2) upper sidebands near the carrier;
3) low-frequency component near dc.

The latter creates phase noise by up-conversion.

Our experience in numerical simulation of phase noise enables us to give an order of magnitude of the influence of the low-frequency component near dc of the white noise source I_n. Practically speaking, it generally plays a lower role than the RF sidebands. Thus, a more accurate empirical expression of

$S_{\Delta\varphi_{\rm out}}$ taking into account the three components can be written as

$$S_{\Delta\varphi_{\rm outTOTAL}} = K S_{\Delta\varphi_{\rm out}} \tag{45}$$

with $1 < K \leq 2$ depending on the up-conversion factor.

Introducing now the $1/f$ component defined by the frequency corner f_c of the noise source I_n results in a frequency corner of $S_{\Delta\varphi_{\rm out}}$, i.e., $f_{c\Delta\varphi}$

$$f_{c\Delta\varphi} = \frac{f_c}{K}. \tag{46}$$

This reduction of the phase noise frequency corner as compared to that of the noise source alone is always observed in numerical simulations, as well as in experiments.

On the other hand, I_n is defined as an input equivalent noise source. In order to take into account transistor noise sources localized at the output port of the transistor, e.g., the collector shot noise source of a bipolar transistor used in common emitter configuration, the equivalent input noise source must be calculated.

The output noise source I_n gives an equivalent input source

$$\left\langle \left| I_{n_{\rm input}} \right|^2 \right\rangle = \frac{\left\langle \left| I_{n_{\rm out}} \right|^2 \right\rangle}{|A_0|^2} \tag{47}$$

where A_0 is the chain-matrix element.

The resulting input phase noise due to the RF sidebands of $I_{n_{\rm out}}$ can be obtained from (6) and (47) as follows:

$$S_{\Delta\varphi_{\rm in}} = 2\, \frac{\overline{\left\langle \left| I_{n_{\rm out}} \right|^2 \right\rangle}}{|V_{01}|^2} \left| \frac{1}{C_0} \right|^2. \tag{48}$$

The input phase noise due to $I_{n_{\rm out}}$ is independent of A_0.

This conclusion is effectively corroborated by numerical simulations of the output phase noise as a function of A_0 for fixed ω_0, $Q_{\rm Loscill}$, C_0, and C_I'.

IX. Conclusion

A detailed analysis of the output phase noise of feedback oscillators has been performed. The results have been compared to the Leeson formula. The hidden parameters included in the formula have been set off and highlighted in (19) and (20), leading to a new formulation better suited to design purposes.

The expression of the loaded Q factor of a feedback oscillator and the definition of the input phase-noise spectral density have been detailed. The new formulations can be easily applied to all types of feedback oscillators including lumped elements and transmission lines.

The agreement obtained by comparing the analytical calculations and numerical simulations performed with a realistic nonlinear transistor model on a nonlinear frequency-domain simulator confirms the validity of the proposed formulation.

Finally, it must be noted that the proposed analysis can be directly extended to oscillator circuits with nonreciprocal elements such as gyrators, isolators, etc. included in the feedback path, and for which $\Delta_{\rm chain} = AD - BC \neq 1$.

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References

[1] D. B. Leeson, "A simple model of feedback oscillator noise spectrum," *Proc. IEEE*, vol. 54, pp. 329–330, Feb. 1966.
[2] M. Odyniec, *RF and Microwave Oscillator Design*. Boston, MA: Artech House, 2002.
[3] H. A. Haus and R. B. Adler, "Invariants of linear noisy networks," in *IRE Conv. Rec.*, 1956, pp. 53–67.
[4] R. Q. Twiss, "Nyquist's and Thevenin's theorems generalized for non reciprocal linear networks," *J. Appl. Phys.*, vol. 26, no. 5, pp. 599–602, May 1955.
[5] P. Penfield, "Circuit theory of periodically driven non linear systems," *Proc. IEEE*, vol. 54, pp. 266–280, Feb. 1966.
[6] W. Robins, *Phase Noise in Signal Sources*. London, U.K.: Perigrinus, 1982.
[7] D. Scherer, "Learn about low-noise design," *Microwaves*, pp. 116–122, Apr. 1979.
[8] F. X. Kaertner, "Analysis of white and $f^{-\alpha}$ noise in oscillators," *Int. J. Circuit Theory Applicat.*, vol. 18, pp. 485–519, 1990.
[9] H. W. Bode, *Network Analysis and Feedback Amplifier Design*. New York: Van Nostrand, 1945.
[10] M. Ohtomo, "Experimental evaluation of noise parameters in Gunn and Avalanche oscillators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 425–437, July 1972.
[11] A. P. S. Khanna, *Dielectric Resonators*, D. Kajfez and P. Guillon, Eds. Dedham, MA: Artech House, 1986.
[12] J. Obregon, M. Prigent, J. C. Nallatamby, M. Camiade, D. Rigaud, and R. Quéré, "Key aspects of modeling and design of low phase noise microwave transistor oscillators," presented at the IEEE MTT-S Int. Microwave Symp. Workshop, Boston, MA, June 2000.

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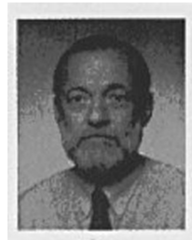

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